ExamPath

CLASS 9 MATHEMATICS · FBISE · UNIT 9 NOTES

Unit 9: Geometry and Polygons

Short notes on reasoning and proof, similar figures, regular polygons, interior and exterior angles, perimeter and area, and loci.

Reasoning | Similar figures | Polygons | Exterior angles | Locus

1. Reasoning
2. Statements and proofs
3. Similar figures
4. Areas and volumes
5. Regular polygons
6. Interior angles
7. Exterior angles
8. Perimeter and area
9. Locus basics
10. Locus applications
11. Common mistakes
12. Quick check

1 · REASONING

Inductive and deductive reasoning

Inductive

Specific → general

Spot a pattern in a few cases and guess a general rule. The conclusion is probable, not proved.

Deductive

General → specific

Apply a rule already known to be true to one case. The conclusion is certain.

Deductive example. All squares have four equal sides. *ABCD* is a square. So *ABCD* has four equal sides.

Inductive trap. 3, 5 and 7 are prime, so "all odd numbers are prime"? But 9 = 3 × 3. One **counterexample** disproves a conjecture, while many examples never prove it.

2 · MATHEMATICAL LANGUAGE

Statements, axioms and theorems

Statement. A sentence that is either true or false. $x + 3 = 7$ is an **open sentence**, because its truth depends on *x*.

Axiom (postulate) is assumed true without proof. A **conjecture** is believed true but not yet proved.

Theorem is proved from accepted statements. A **corollary** follows easily from a theorem. The **converse** of "if *p* then *q*" is "if *q* then *p*".

Proof. A chain of logical steps. Each step must rest on an axiom, a definition or an earlier theorem, so the conclusion is guaranteed.

A converse may fail. "If a polygon is regular, all its sides are equal" is true. Its converse is false: a rhombus has equal sides but is not regular.

3 · SIMILAR FIGURES

Same shape, not necessarily same size

Figures are **similar** when they have the same shape but not the same size. Corresponding angles are equal and corresponding sides are in the same ratio.

Always similar: any two squares. **Not always:** two rectangles. For 2 × 3 and 4 × 9, the ratios 2/4 and 3/9 differ.

Triangles. If two triangles are similar, their corresponding sides are **proportional**. Equal corresponding angles are enough to make them similar.

Example. Triangle ABC has sides 3, 4, 5 cm and is similar to PQR with PQ = 6 cm. The scale factor is 6/3 = 2, so QR = 2 × 4 = **8 cm**.

Congruent vs similar. Congruent triangles are similar (ratio 1), but similar triangles need not be congruent.

Parallel line. In equilateral triangle ABC, if $DE \parallel BC$, corresponding angles give $\angle D = \angle E = 60°$, so ADE is also equilateral.

4 · SIMILAR FIGURES AND SIZE

Squares for area, cubes for volume

Area

ratio of areas = (ratio of lengths)^2

Volume

ratio of volumes = (ratio of lengths)^3

Doubling. Double a square's side and its area becomes 4 times. Double a cube's edge and its volume becomes 8 times.

Model. Scale 1 : 20. A model roof of area 3 m^2 means an actual roof of $3 \times 20^2 =$ **1200 m^2**.

Tank. A 40 L tank, and a similar tank 3 times as tall: $40 \times 3^3 =$ **1080 L**.

Pyramids. Lengths 14 cm and 10 cm, larger surface area 392 cm^2. Smaller: $392 \times (10/14)^2 = 392 \times 25/49 =$ **200 cm^2**.

Cylinders. Radii 3 ft and 2 ft, smaller volume 40 ft^3. Larger: $40 \times (3/2)^3 =$ **135 ft^3**.

If the volume ratio is 8 : 27, the length ratio is the cube root, **2 : 3**.

5 · POLYGONS

Regular and irregular

A **polygon** is a closed figure made of straight segments. The fewest sides it can have is 3.

Regular: all sides equal **and** all interior angles equal. Examples: equilateral triangle, square.

Irregular: at least one side or one angle differs. A rectangle (not a square) has equal angles but unequal sides. A rhombus has equal sides but unequal angles. Neither is regular.

Perimeter. For a regular polygon with n sides of length s: $P = n \times s$.

6 · INTERIOR ANGLES

Split the polygon into triangles

$$\text{Sum} = (n - 2) \times 180°$$

Joining one vertex to the others makes $n - 2$ triangles, each with angle sum 180°. A quadrilateral gives 2 triangles, so its angle sum is 360°.

Diagonals. A polygon with n sides has $n(n - 3)/2$ diagonals. A hexagon has 6 × 3 / 2 = 9.

Each angle of a regular polygon = (n − 2) × 180° ÷ n. Nonagon: 7 × 180° ÷ 9 = **140°**. Hexagon: **120°**.

Sum for 12 sides. 10 × 180° = **1800°**.

Missing angle. In a hexagon the angles are 90°, 155°, 112°, 140°, 98° and x. Their sum is 720°, so $x = 720 - 595 =$ **125°**.

Not every sum is possible. 9 right angles = 810°. Then n − 2 = 810 ÷ 180 = 4.5, which is not a whole number, so no polygon has that angle sum.

7 · EXTERIOR ANGLES

Always 360° in total

At each vertex, interior + exterior = 180°, because they form a linear pair.

Sum of exterior angles (one per vertex) = **360°** for every polygon, whatever the number of sides.

Regular polygon: each exterior angle = $360°/n$.

Find the sides. Exterior angle 120°: n = 360/120 = 3, an equilateral triangle. Exterior angle 45°: n = 8. Interior angle 156°: exterior 24°, so n = 15.

Twice as big. If the interior sum is twice the exterior sum: $(n - 2) \times 180° = 2 \times 360°$, so $n - 2 = 4$ and n = 6, a hexagon.

Unknown angles. Six exterior angles $(x+4)°$, $(3x-4)°$, $(7x-3)°$, $(2x+3)°$, $(8x-1)°$, $(9x+1)°$ add to 360°: $30x$ = 360, so x = 12 and the angles are 16°, 32°, 81°, 27°, 95°, 109°.

Impossible. An exterior angle of 50° would give n = 360 ÷ 50 = 7.2 sides. A regular polygon cannot have that.

8 · REGULAR POLYGONS IN USE

Perimeter and area

Apothem *a* is the perpendicular distance from the centre to a side. It is also the radius of the inscribed circle.

$P = n \times s$ and $A = \frac{1}{2} P a$

The polygon splits into n triangles of base s and height a, so $A = n \times \frac{1}{2}sa = \frac{1}{2}(ns)a$.

Water tank base. Regular pentagon, side 8 ft, apothem 7 ft. $P = 40$ ft, $A = \frac{1}{2} \times 40 \times 7 =$ **140 ft^2**.

Fencing. Regular hexagonal plot, side 10 m, fencing Rs 160 per metre. $P = 60$ m, cost = 60 × 160 = **Rs 9,600**.

Square room. Perimeter 48 ft means side 12 ft. With height 10 ft, the four walls have area 4 × 12 × 10 = **480 ft^2**.

9 · LOCUS

A path that follows a rule

A **locus** is the set of all points that satisfy a given condition. The plural is **loci**. Every point on the locus satisfies the condition, and every point satisfying it lies on the locus.

Fixed distance from a point. The locus is a **circle** in a plane (a sphere in three-dimensional space).

Fixed distance *a* from a line. A **pair of parallel lines**, one on each side of the line, each at distance *a*.

Equidistant from two parallel lines. A parallel line **halfway** between them.

Equidistant from two points *A* and *B*. The **perpendicular bisector** of *AB*, so *PA* = *PB* for every point *P* on it.

10 · LOCUS APPLICATIONS

Bisectors and the circumcircle

Equidistant from two intersecting lines. A pair of lines that bisect the angles formed by the two lines. Every point on a bisector is equidistant from both lines.

Three non-collinear points *A*, *B*, *C*. The perpendicular bisectors of *AB* and *BC* meet at *O*, so $OA = OB = OC$. The circle through the three points is the **circumcircle** and *O* is the **circumcentre**.

Concentric circles. Points that stay at fixed distances from the same point *O* trace circles with the same centre.

Two isosceles triangles on one base. *XAB* and *YAB* share base *AB*. Since *XA* = *XB* and *YA* = *YB*, both *X* and *Y* lie on the perpendicular bisector of *AB*, so line *XY* bisects *AB* at right angles.

11 · COMMON MISTAKES

Five traps to avoid

Doubling the side doubles the area. It makes it 4 times larger. Volume becomes 8 times.

Using (*n* – 2) × 180° for exterior angles. The exterior angles always add to 360°.

Calling a rhombus or rectangle regular. A regular polygon needs both equal sides and equal angles.

Treating a locus from two points as a circle. Equal distances from two points give a straight line, the perpendicular bisector.

Assuming a conjecture is proved by examples. One counterexample is enough to disprove it.

12 · QUICK CHECK

Try these, then check your answers

What is each interior angle of a regular hexagon?

Answer: $(6 - 2) \times 180° \div 6 = 120°$.

How many sides has a regular polygon with exterior angle 45°?

Answer: $360° \div 45° = 8$ sides.

Two similar solids have lengths in the ratio 2 : 3. What is the ratio of their volumes?

Answer: $2^3 : 3^3 = 8 : 27$.

Free practice notes from ExamPath (exampath.org). You may print and share them for study. These notes are written independently and are not an official FBISE document. Practise Unit 9 with timed MCQ tests at exampath.org.