ExamPath

CLASS 9 MATHEMATICS · FBISE · UNIT 8 NOTES

Unit 8: Geometry of Straight Lines

Short notes on gradient and inclination, the different forms of a line's equation, parallel and perpendicular lines, angles between lines and intersections.

Gradient | Forms of equations | Parallel and perpendicular | Angle between lines | Intersections

1. Inclination and gradient
2. Gradient from two points
3. Parallel and perpendicular
4. Equations: basic forms
5. Equations: more forms
6. The general form
7. Finding an equation
8. Angle between lines
9. Intersection and families
10. Lines in real life
11. Common mistakes
12. Quick check

1 · INCLINATION AND GRADIENT

How steep is a line?

Inclination θ is the angle the line makes with the **positive x-axis**, measured anticlockwise.

Gradient (slope). $m = \tan\theta$. For example, inclination 45° gives $m = \tan 45° = 1$.

Horizontal line: $\theta = 0°$, so $m = 0$.
Vertical line: $\theta = 90°$, so the run is zero and the gradient is **undefined**.

Sign. A line rising from left to right has $m > 0$ ($0° < \theta < 90°$). A negative gradient means $90° < \theta < 180°$.

2 · GRADIENT FROM TWO POINTS

Rise over run

$$m = (y_2 - y_1)/(x_2 - x_1)$$

$(2, 6)$ and $(5, 8)$: $m = (8 - 6)/(5 - 2) =$ **2/3**

Either point can come first, as long as both differences are taken in the same order.

Do not mix the order. For (1, 2) and (3, 6), writing (1 − 3)/(6 − 2) is wrong. The correct value is (6 − 2)/(3 − 1) = 2.

Real life. A hill with gradient 1/25 rises 1 unit for every 25 units of horizontal distance.

3 · PARALLEL AND PERPENDICULAR

Compare the gradients

Parallel

Equal gradients: $m_1 = m_2$. They make the same angle with the x-axis.

Perpendicular

Product −1: $m_1 \times m_2 = -1$.

Examples. $2/3 \times (-3/2) = -1$, so these slopes are perpendicular. So are 1 and −1.

Slip. The line perpendicular to slope 2 does not have slope −2, because 2 × (−2) = −4. It has slope $-1/2$.

Collinear points. If the slopes of *AB* and *BC* are equal, the lines share point *B* and are the same line. For (1, 1), (2, 2), (3, 4) the slopes are 1 and 2, so the points are not collinear.

4 · EQUATIONS OF A LINE

Horizontal, vertical and slope forms

Vertical line: $x = a$, parallel to the y-axis. **Horizontal line:** $y = b$, parallel to the x-axis. The horizontal line through (2, 3) is $y = 3$. The x-axis is $y = 0$.

Slope-intercept form: $y = mx + c$, where *c* is the y-intercept. For $y = -2x + 5$: *m* = −2 and *c* = 5.

Point-slope form: $y - y_1 = m(x - x_1)$, for a line through (x_1, y_1) with slope *m*.

Point-slope form cannot describe a **vertical** line, because its slope is undefined. Use $x = a$.

5 · EQUATIONS OF A LINE

Two-point, intercept and normal forms

Two-point form. Through (2, −1) and (5, 8): slope = 9/3 = 3, so $y + 1 = 3(x - 2)$, which gives **$y = 3x - 7$**.

Two-intercept form. $x/a + y/b = 1$ for a line through (*a*, 0) and (0, *b*). With intercepts 3 and 5: $x/3 + y/5 = 1$, and multiplying by 15 gives **$5x + 3y = 15$**.

Normal form. $x \cos\alpha + y \sin\alpha = p$, where *p* is the perpendicular distance from the origin to the line, so *p* is positive.

6 · THE GENERAL FORM

$ax + by + c = 0$

a and b must not both be zero. If both were, the equation would become $c = 0$, which is not a line.

Reading it off ($b \neq 0$): slope $-a/b$, y-intercept $-c/b$. For $a \neq 0$ the x-intercept is $-c/a$.

$3x - 4y + 8 = 0$: slope = −3/(−4) = **3/4**.

$2x + 3y - 6 = 0$: the slope is **−2/3**, not 2/3. Writing $y = -(2/3)x + 2$ shows it.

If $b = 0$, as in $5x - 6 = 0$, then $x = -c/a$, a vertical line. A line through the origin has **no constant term**, like $y = 3x$.

7 · WORKED EXAMPLES

Use a point and a condition

Perpendicular line. Find the line through (2, −3) perpendicular to a line of slope 2/3.
The slope is −3/2. So $y + 3 = -(3/2)(x - 2)$, giving $2y + 6 = -3x + 6$, that is **$3x + 2y = 0$**.

Finding a constant. The line $3x + ky - 11 = 0$ passes through (1, 2). Substitute: $3 + 2k - 11 = 0$, so **$k = 4$**.

Substituting a point into an equation tests whether the point lies on the line.

8 · ANGLE BETWEEN TWO LINES

From slope m_1 to slope m_2

$$\tan\theta = (m_2 - m_1)/(1 + m_1 m_2)$$

Slopes 1/2 and 3: $\tan\theta = (3 - 1/2)/(1 + 3/2) = (5/2)/(5/2) = 1$, so **$\theta = 45°$**.

Slopes 0 and √3: $\tan\theta = \sqrt{3}/1$, so **$\theta = 60°$**.

Parallel lines: $m_1 = m_2$ makes the top zero, so $\theta = 0°$.

Perpendicular lines: $m_1 m_2 = -1$ makes the denominator zero, so $\tan\theta$ is undefined and $\theta = 90°$.

9 · INTERSECTING LINES

Where two lines meet

Two lines with **different slopes** meet at exactly one point.

$x + y = 7$ and $x - y = 1$. Adding gives $2x = 8$, so $x = 4$ and $y = 3$. The point is **(4, 3)**.

Family of lines. Every line of the form $(a_1x + b_1y + c_1) + k(a_2x + b_2y + c_2) = 0$ passes through the intersection point of the two original lines, because that point makes both brackets zero.

10 · REAL-LIFE LINES

Slope is a rate, intercept is a start

Taxi. $C = 25d + 100$. The slope 25 is the cost per kilometre. The y-intercept 100 is the fixed charge at $d = 0$.

Temperature. $F = (9/5)C + 32$. The slope 9/5 is the change in F for each 1° change in C.

Meter reading. It shows 50 units and 12 units are used per day. $50 + 12d = 410$ gives $12d = 360$, so after **30 days**.

11 · COMMON MISTAKES

Five traps to avoid

Perpendicular slope. It is the negative reciprocal, not just the negative.

Wrong sign for the slope of $ax + by + c = 0$. It is $-a/b$.

Reversing the order in one difference only in the gradient formula.

Giving a vertical line a gradient. Its gradient is undefined, and its equation is $x = a$.

Forgetting what equal slopes mean. Two lines with equal slopes are parallel. If they also share a point, they are the same line.

12 · QUICK CHECK

Try these, then check your answers

Find the gradient of the line through $(2, 6)$ and $(5, 8)$.

Answer: (8 − 6)/(5 − 2) = 2/3.

What is the slope of $3x - 4y + 8 = 0$?

Answer: $-a/b = -3/(-4) = 3/4$.

Find the angle from the line of slope 0 to the line of slope √3.

Answer: tan $\theta = \sqrt{3}$, so $\theta = 60°$.

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