**ExamPath**

CLASS 9 MATHEMATICS · FBISE · UNIT 7 NOTES

# Unit 7: Coordinate Geometry

Short notes on the coordinate plane, the distance and mid-point formulas, collinear points, and identifying triangles and quadrilaterals.

Coordinate plane | Distance formula | Mid-point | Collinear points | Triangles and quadrilaterals


1. The coordinate plane
2. Distance on a line
3. Distance formula
4. Collinear points
5. Mid-point formula
6. Using the mid-point
7. Types of triangles
8. Types of quadrilaterals
9. Common mistakes
10. Quick check


1 · THE COORDINATE PLANE

## Every point has an address

A point is written $(x, y)$. $x$ is the **abscissa** (first coordinate) and $y$ is the **ordinate** (second coordinate). The axes meet at the **origin** $(0, 0)$.

**On the axes.** Every point on the x-axis has $y = 0$, so it looks like $(x, 0)$. Every point on the y-axis has $x = 0$. So $(0, -5)$ lies on the y-axis.

**Order matters.** $(2, 3)$ and $(3, 2)$ are different points. They are the same only when $x = y$.

**Quadrant I** $(+, +)$

**Quadrant II** $(-, +)$ e.g. (-3, 4)

**Quadrant III** $(-, -)$

**Quadrant IV** $(+, -)$

2 · DISTANCE ON A NUMBER LINE

## Subtract, then take the absolute value

$$d(A, B) = |b - a|$$

**Example.** Points -3 and 5: $|5 - (-3)| = |5 + 3| = 8$.

Reversing the points only changes the sign inside the bars, so $d(A, B) = d(B, A)$. A distance is never negative.

Two points with the **same *y*-coordinate** lie on a horizontal line, so only the *x*-coordinates matter.

3 · THE DISTANCE FORMULA

## Pythagoras on the plane

The horizontal and vertical differences are the two legs of a right-angled triangle. The distance is its hypotenuse.

$$d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$$

$(0, 0)$ to $(3, 4)$: $\sqrt{(9 + 16)} =$ **5**

$(2, -1)$ to $(14, 4)$: $\sqrt{(12^2 + 5^2)} = \sqrt{169} =$ **13**

$(5, 3)$ to $(-1, -5)$: $\sqrt{(36 + 64)} =$ **10**. And $(-8, 15)$ from the origin: $\sqrt{(64 + 225)} = 17$.

**Either point can come first.** Squaring removes the sign, so reversing the points gives the same result.

**Unknown coordinate.** The distance between $(1, 2)$ and $(4, k)$ is 5 and $k > 2$. Then $9 + (k - 2)^2 = 25$, so $k - 2 = \pm 4$. Since $k > 2$, ***k* = 6**.

4 · COLLINEAR POINTS

## When three points lie on one line

If *B* lies on the segment *AC*, the two parts add up to the whole: $AB + BC = AC$.

**Example.** *AB* = √5, *BC* = √5 and *AC* = 2√5. Since √5 + √5 = 2√5, the three points are on one line.

**Not collinear.** If $AB + BC > AC$, the points form a triangle. For *AB* = 3, *BC* = 4 and *AC* = 5, the sum 7 is greater than 5, so they are not collinear.

5 · THE MID-POINT

## Average the coordinates

$$M = ((x_1 + x_2)/2, (y_1 + y_2)/2)$$

$(4, 2)$ and $(10, 8)$: $((4 + 10)/2, (2 + 8)/2) =$ **(7, 5)**

$(-3, 4)$ and $(5, -2)$: $((-3 + 5)/2, (4 - 2)/2) =$ **(1, 1)**

**Finding an end point.** *M*(2, 3) is the mid-point of *A*(−1, 5) and *B*(*x*, *y*).
(−1 + *x*)/2 = 2 gives *x* = 5. (5 + *y*)/2 = 3 gives *y* = 1. So ***B* = (5, 1)**.

The mid-point splits a segment into two equal parts, so $LM : MN = 1 : 1$.

6 · MID-POINT IN ACTION

## Centres and bisectors

**Centre of a circle.** The centre is the mid-point of any diameter. For a diameter with ends $(4, 5)$ and $(6, 9)$, the centre is $(5, 7)$.

**Bisectors.** Every bisector of a segment cuts it into two equal parts, so all bisectors pass through its mid-point.

**Bisecting each other.** Two segments that bisect each other are cut at the same point, their common mid-point. That is why the diagonals of a parallelogram share a mid-point.

**Mid-point theorem.** The segment joining the mid-points of two sides of a triangle is parallel to the third side and half its length.

7 · TRIANGLES FROM COORDINATES

## Find the three sides, then decide

**Isosceles:** two equal sides. **Equilateral:** three equal sides.

**Right-angled:** the squares of two sides add up to the square of the third (converse of Pythagoras theorem).

**Example.** Vertices $(0, 0)$, $(4, 0)$, $(2, 3)$. The sides are 4, $\sqrt{13}$ and $\sqrt{13}$. Two sides are equal, so it is isosceles. It is not right-angled, because $4^2 = 16$ but $13 + 13 = 26$.

**Perimeter.** Vertices $(0, 0)$, $(3, 0)$, $(0, 4)$. The sides are 3, 4 and $\sqrt{(9 + 16)} = 5$, so the perimeter is **12**.

8 · QUADRILATERALS FROM COORDINATES

## What to show for each shape

**Parallelogram.** Opposite sides parallel and equal. The diagonals bisect each other.

**Rectangle.** A parallelogram with **equal diagonals**. The diagonals of a rectangle are equal and bisect each other.

**Rhombus.** It is enough to show that all four sides are equal.

**Trapezium.** Exactly one pair of parallel sides.

9 · COMMON MISTAKES

## Four traps to avoid

**Adding the differences.** The distance between (1, 2) and (4, 6) is not 3 + 4 = 7. It is the hypotenuse, $\sqrt{(3^2 + 4^2)} = 5$.

**Forgetting to take the square root** in the distance formula.

**Mixing up the coordinates.** The abscissa is $x$ and the ordinate is $y$.

**Keeping both roots** when a condition rules one out, such as $k > 2$ above.

10 · QUICK CHECK

## Try these, then check your answers

Find the mid-point of $(4, 2)$ and $(10, 8)$.

**Answer:** (7, 5).

Find the distance between $(2, -1)$ and $(14, 4)$.

**Answer:** 13, since $\sqrt{(144 + 25)} = 13$.

In which quadrant is $(-3, 4)$?

**Answer:** Quadrant II: negative x, positive y.

Free practice notes from ExamPath (exampath.org). You may print and share them for study. These notes are written independently and are not an official FBISE document. Practise Unit 7 with timed MCQ tests at exampath.org.