ExamPath

CLASS 9 MATHEMATICS · FBISE · UNIT 4 NOTES

Unit 4: Factorization and Algebraic Manipulation

Short notes on factorization, identities, HCF and LCM, square roots and algebraic fractions.

Identities | Trinomials | Cubes | HCF and LCM | Algebraic fractions

1. Factorization basics
2. Splitting the middle term
3. Cubes
4. Harder factorization
5. HCF and LCM
6. HCF and LCM relations
7. Square roots
8. Algebraic fractions
9. Adding fractions
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1 · FACTORIZATION

Write an expression as a product

Factorization is the opposite of multiplying out. For example, $5x$ and $3xy$ are factors of $15x^2y$ because their product gives it.

Difference of two squares

$a^2 - b^2 = (a + b)(a - b)$

So $x^2 - 4 = (x + 2)(x - 2)$. Note that $x - 4$ is not a factor.

Perfect square trinomials

$a^2 + 2ab + b^2 = (a + b)^2$ and $a^2 - 2ab + b^2 = (a - b)^2$

Check the middle term. In $4x^2 + 12x + 9$, the middle term is $2 \times (2x) \times 3 = 12x$, so it equals $(2x + 3)^2$.

2 · TRINOMIALS

Split the middle term

For $x^2 + px + q$, find two numbers whose **product is *q*** and whose **sum is *p***.

$x^2 + 5x + 6$: the numbers are 2 and 3.
$= (x + 2)(x + 3)$

$x^2 - 7x + 12$: the product 12 is positive, so both numbers have the same sign. The sum is negative, so both are negative: −3 and −4.
$= (x - 3)(x - 4)$

$2x^2 - 5x - 3$: split $-5x$ as $-6x + x$.
$= 2x(x - 3) + 1(x - 3) = (2x + 1)(x - 3)$

$3x^2 + 11x + 6$: split $11x$ as $9x + 2x$.
$= 3x(x + 3) + 2(x + 3) = (3x + 2)(x + 3)$

3 · CUBES

Sum and difference of cubes

Sum of cubes
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Difference of cubes
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Cube of a sum
$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Example. $x^3 + 27 = x^3 + 3^3 = (x + 3)(x^2 - 3x + 9)$

Careful. $(x - y)^3 \neq x^3 - y^3$. The left side expands to $x^3 - 3x^2y + 3xy^2 - y^3$.

4 · CLEVER METHODS

Turn it into something you know

Group into a square. $a^2 + 2ab + b^2 - c^2 = (a+b)^2 - c^2 = (a+b+c)(a+b-c)$

Add and subtract a term. $a^4 + 4b^4 = (a^2 + 2b^2)^2 - (2ab)^2 = (a^2 + 2ab + 2b^2)(a^2 - 2ab + 2b^2)$

Add and subtract a^2b^2. $a^4 + a^2b^2 + b^4 = (a^2 + b^2)^2 - (ab)^2 = (a^2 + ab + b^2)(a^2 - ab + b^2)$

Substitute. For $(x^2 + 3x - 4)(x^2 + 3x + 5) + 8$, put $a = x^2 + 3x$.
$(a-4)(a+5) + 8 = a^2 + a - 12 = (a+4)(a-3)$.
Put back a: $(x^2 + 3x + 4)(x^2 + 3x - 3)$.

5 · HCF AND LCM

Factorize first, then compare

HCF

Take the **common** factors, each with its **lowest** power.

LCM

Take **every** factor, each with its **highest** power.

Monomials. $12x^2y$ and $18xy^2$: HCF of 12 and 18 is 6, lowest powers x and y, so the HCF is $6xy$.
$6x^2y$ and $9xy^3$: LCM of 6 and 9 is 18, highest powers x^2 and y^3, so the LCM is $18x^2y^3$.

Polynomials. $x^2 - x - 6 = (x-3)(x+2)$ and $x^2 + x - 2 = (x+2)(x-1)$.
LCM = $(x-3)(x+2)(x-1)$.

Powers matter. $a^2 - b^2 = (a+b)(a-b)$ and $(a-b)^2$ have HCF $a - b$.

6 · USEFUL FACTS

HCF and LCM together

$$\text{HCF} \times \text{LCM} = P \times Q$$

If the HCF of P and Q is 1, then $\text{LCM} = P \times Q$.

Multiplying one polynomial by a non-zero constant does not change the HCF. If the HCF of P and Q is $x^2 + 2x$, the HCF of $5P$ and Q is the same.

Division method for HCF. You may multiply by a **non-zero constant** to avoid fractions, because that adds no new polynomial factor.

Do **not** multiply by a **variable**. It can bring in a new common factor and give a wrong HCF.

7 · SQUARE ROOTS

Square root of an expression

If $P = Q \times Q$, then Q is a square root of P.

Both $+Q$ and $-Q$ work, so the square roots are $\pm Q$.

Example. $4a^2 + 12ab + 9b^2 = (2a + 3b)^2$, so the square roots are $\pm(2a + 3b)$.

Method. Factorize into a perfect square, then take one factor.

8 · ALGEBRAIC FRACTIONS

Simplify, multiply and divide

The denominator can never be zero. For P/Q, we need $Q \neq 0$.

Factorize, then cancel. $(x^2 - 9)/(x - 3) = (x + 3)(x - 3)/(x - 3) = x + 3$, for $x \neq 3$.

Dividing means multiplying by the reciprocal.
$(x^2 - 1)/(x + 2) \div (x - 1)/(x^2 - 4) = (x + 1)(x - 1)/(x + 2) \times (x + 2)(x - 2)/(x - 1) =$ **$(x + 1)(x - 2)$**

For products, factorize all numerators and denominators first, so you can cancel to the most reduced form.

9 · ADDING AND SUBTRACTING

Use the LCM as the common denominator

With unlike denominators, first find the **LCM of the denominators** and rewrite each fraction.

$3/(x - 1) + 2/(x + 1)$
The LCM is $x^2 - 1$.
$= (3(x + 1) + 2(x - 1))/(x^2 - 1) =$ **$(5x + 1)/(x^2 - 1)$**

Order of operations. DMAS: division, multiplication, addition, subtraction.

Complex fraction. A fraction whose numerator, denominator or both contain algebraic expressions.

10 · COMMON MISTAKES

Five traps to avoid

Cancelling terms. $(x+2)/x \neq 2$. Only common **factors** cancel. Here the fraction equals $1 + 2/x$.

Missing middle term. $x^2 + 9 \neq (x+3)^2$, because $(x+3)^2 = x^2 + 6x + 9$.

Forgetting the restriction. After simplifying $(x^2 - 9)/(x - 3)$ to $x + 3$, remember $x \neq 3$, since the original fraction is undefined there.

Wrong sign in a cube factor. The bracket for $a^3 + b^3$ has $a^2 - ab + b^2$, the opposite sign to the one in the first factor.

Multiplying by a variable in the division method for HCF. Use only a non-zero constant.

11 · QUICK CHECK

Try these, then check your answers

Factorize $x^2 + 5x + 6$.

Answer: $(x + 2)(x + 3)$, since 2 × 3 = 6 and 2 + 3 = 5.

Find the HCF of $12x^2y$ and $18xy^2$.

Answer: $6xy$.

Simplify $(x^2 - 9)/(x - 3)$, for $x \neq 3$.

Answer: $x + 3$.

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