ExamPath

CLASS 9 MATHEMATICS · FBISE · UNIT 3 NOTES

Unit 3: Sets and Relations

Short notes on sets, set operations, counting with Venn diagrams, ordered pairs, Cartesian products and relations.

Sets | Operations | Counting | Ordered pairs | Cartesian product | Relations

1. What a set is
2. Kinds of sets
3. Union and intersection
4. Difference and complement
5. Useful laws
6. Counting with sets
7. Ordered pairs
8. Cartesian product
9. Relations
10. Common mistakes
11. Quick check

1 · SETS

A set is a well-defined collection

Well-defined means we can always say whether an object belongs. “Stones weighing less than 1 kg” is a set. “Beautiful flowers” is not, because opinions differ.

Distinct elements. Repeats are counted once, so $\{2, 3, 3, 4\}$ is written $\{2, 3, 4\}$.

Three ways to write a set

Descriptive: the set of even numbers less than 10

Tabular (roster): $\{2, 4, 6, 8\}$

Set builder: $\{x \mid x \in Z\}$, read “the set of all x such that x belongs to Z”

N natural | W whole | Z integers | Q rational

2 · KINDS OF SETS

Empty, subset, disjoint, overlapping

Empty set { } or φ has no elements. **{0} is not empty**: it has one element, 0.

Subset. $A \subseteq B$ when every element of A is also in B.

Disjoint sets have no common element, so $A \cap B = \varphi$.

Overlapping sets share at least one element, and neither contains the other. For example $\{1, 3, 5\}$ and $\{3, 5, 7\}$.

Universal set *U*. In a Venn diagram it is the rectangle, and other sets are closed shapes inside it.

3 · OPERATIONS

Union and intersection

$A = \{1, 2, 3, 4, 5\}$ $B = \{1, 3, 4, 6, 7\}$

Union $A \cup B$: everything in A or B or both.
$\{1, 2, 3, 4, 5, 6, 7\}$

Intersection $A \cap B$: only what is in both.
$\{1, 3, 4\}$

4 · OPERATIONS

Difference and complement

Difference. $A - B = \{x \mid x \in A \text{ and } x \notin B\}$
If $A = \{1, 2, 3, 4, 5\}$ and $B = \{3, 4, 6, 7, 8\}$, remove 3 and 4 from A: $A - B = \{1, 2, 5\}$.

Complement. $A' = U - A$
If $U = \{1, \ldots, 7\}$ and $A = \{2, 4, 6\}$, then $A' = \{1, 3, 5, 7\}$.

Nothing is shared between A and its complement: $A \cap A' = \varphi$.

If $A \subset B$, then $A - B = \varphi$, because nothing is left after removing B's elements.

5 · LAWS OF SETS

Rules you can rely on

Same set twice. $A \cup A = A$ and $A \cap A = A$

With the empty set. $A \cup \varphi = A$ and $A \cap \varphi = \varphi$

When $A \subset B$. $A \cap B = A$ and $A \cup B = B$

Distributive laws.
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Equal sets. If $A - B = \varphi$ and $B - A = \varphi$, then A ⊆ B and B ⊆ A, so $A = B$.

6 · COUNTING

Do not count the overlap twice

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

The common elements are counted once in $n(A)$ and again in $n(B)$, so one copy is subtracted. For disjoint sets there is no overlap, so $n(A \cup B) = n(A) + n(B)$.

Class of 80. 40 like English, 34 like Maths, 9 like both.
$n(E \cup M) = 40 + 34 - 9 = 65$, so **80 − 65 = 15** like neither.

Survey of 60. 25 watch channel A, 16 watch B, 4 watch both.
$25 + 16 - 4 =$ **37** watch A or B.

7 · ORDERED PAIRS

Order matters

(a, b): *a* is the **first element** and *b* is the **second element**.

$(2, 5) \neq (5, 2)$. Swapping the elements gives a different pair, like two different points.

Equal pairs. $(a, b) = (c, d)$ when $a = c$ and $b = d$. So if $(x, 3) = (2, y)$, then $x = 2$ and $y = 3$.

Example. Find x and y if $(x - 3y, 5x + 1) = (4, 6)$.
From $5x + 1 = 6$, $x = 1$. Then $1 - 3y = 4$ gives **$y = -1$**.

8 · CARTESIAN PRODUCT

Pair every element with every element

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

Example. $A = \{0, 1, 2\}$, $B = \{3, 4\}$
$A \times B = \{(0,3), (0,4), (1,3), (1,4), (2,3), (2,4)\}$, which has **3 × 2 = 6** pairs.

$n(A \times B) = n(A) \times n(B)$, so $n(A \times A) = n^2$.

$A \times B \neq B \times A$ in general: same number of pairs, but written in the opposite order. They are equal only when A = B (non-empty sets).

If $A = \varphi$, no pair can be formed, so $A \times B = \varphi$.

9 · BINARY RELATIONS

A relation is a set of ordered pairs

A **binary relation** from A to B is any subset of $A \times B$.

Domain = set of first elements. **Range** = set of second elements.

Example. $R = \{(2,1), (4,3), (2,2)\}$
Dom R = {2, 4} (the 2 is not repeated). Range = {1, 2, 3}.

Inverse. R^{-1} swaps every pair. If $R = \{(0,0), (2,4), (3,9)\}$, then $R^{-1} = \{(0,0), (4,2), (9,3)\}$.

Counting relations. A set with k elements has 2^k subsets. With $n(A) = 3$ and $n(B) = 2$, $A \times B$ has 6 pairs, so there are **$2^6 = 64$** relations.

Void relation. $R = \varphi$, a relation with no ordered pairs.

10 · COMMON MISTAKES

Five traps to avoid

Calling {0} empty. It has one element. Only { } or φ is empty.

Adding n(A) + n(B) for overlapping sets. Subtract $n(A \cap B)$ once.

Swapping an ordered pair. $(a, 1) \neq (1, a)$.

Repeating elements in a domain. A set lists each element once.

Assuming A × B = B × A. It holds only when A and B are the same set.

11 · QUICK CHECK

Try these, then check your answers

If $U = \{1, \ldots, 7\}$ and $A = \{2, 4, 6\}$, find A'.

Answer: {1, 3, 5, 7}.

If $n(A) = 4$ and $n(B) = 3$, how many relations are in $A \times B$?

Answer: 4 × 3 = 12 pairs, so 2^{12} = 4096 relations.

What is $A \cap B$ if $A \subset B$?

Answer: A itself, since every element of A is already in B.

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